MATH 9144A - ASSIGNMENT 4

(1) Let $F : \mathcal{A} \to \mathcal{B}$ be a right exact functor between abelian categories. Assume $\mathcal{A}$ has enough projectives. Prove that $\mathbb{L}_i F$ restricted to $\mathrm{Ch}_{\geq 0}(\mathcal{A})$ is the left derived functor of the right exact functor $H_0 F$.

(2) Let $G = \mathbb{Z}/p$ and $k = \mathbb{F}_p$. Assume that there exists a finite length kG-resolution C_* of k where each C_n is a finitely generated permutation module. Let D_* denote the sub-complex which in degree n consists of the kG/G summands in C_n. Prove that D_* is a kG-resolution of k. *Hint:* Compare $H_*(G, C)$ and $H_*(G, D)$.

(3) The quadratic group Q_8 is a central extension of the form

$$0 \to \mathbb{Z}/2 \to Q_8 \to \mathbb{Z}/2 \times \mathbb{Z}/2 \to 0.$$

What is the image of $d_2 : E_2^{0,1} \to E_2^{2,0}$ in the Lyndon-Hochschild-Serre spectral sequence in mod 2 coefficients associated to the extension? *Hint:* Consider restricting to subgroups.

(*Bonus*) Let $f : \mathbf{C}^{\mathrm{op}} \to \mathbf{D}^{\mathrm{op}}$ be a functor between small categories. Consider the functor $f_* : \mathbf{Ab}^{\mathbf{D}^{\mathrm{op}}} \to \mathbf{Ab}^{\mathbf{C}^{\mathrm{op}}}$ defined by $f_*(F) = F \circ f$. Construct a spectral sequence similar to Leray spectral sequence, except that in this case you need to use a right adjoint.